The surface film of substance A (probably vitamin D), in particular, resembles that of oxycholesterylene ($\Delta^{4,6}$ cholestadiene-8one). The ultra-violet absorption spectra of the two substances are very closely similar, and of intensity about twice that of ergosterol. Oxycholesterylene is, however, devoid of antirachitic properties, both before and after irradiation.

The chemical changes leading to ketone formation and their bearing on the production of vitamin D are discussed. The lability of the hydrogen of the CH(OH) group is considered to be the controlling factor of the changes induced by ultra-violet irradiation of ergosterol.

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Wave Mechanics and the Dual Aspect of Matter and Radiation.

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§ 1. Recent researches in experimental and theoretical Physics strongly indicate that both matter and radiation possess a dual aspect exhibiting, on the one hand, the properties of undulatory phenomena extensible throughout the space-time continuum, and on the other, those of "parcel" or particle phenomena concentrated in small regions, the so-called "world lines" of particles. The wave mechanics of de Broglie and Schrödinger is an attempt to bridge the gulf, so to speak, between matter and radiation; and the question naturally arises as to whether these two apparently distinct entities may not represent the same, or at least similar, world conditions.

Let (Ox, Oy, Oz) be a rectangular system of axes at rest relative to a material observer, and consider the motion of an electron relative to the observer. First, let the electron be conceived as a particle, *i.e.*, as a region of the observer's space bounded by a movable surface. It will be necessary for the following analysis to assign to this surface a definite form, and I shall choose the form of a Heaviside ellipsoid associated with the Lorentz electron, since this is the only form that is known to satisfy the geometrical conditions of the space-time continuum. For simplicity, let it be assumed that the centre of the electron

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moves with uniform velocity $c\beta < c$, where c is the velocity of light *in vacuo*, along a line Ox' whose direction cosines are (l, m, n). If (Ox', Oy', Oz') is a rectangular system of axes then every point on the surface of the electron will satisfy the equation

$$(x' - c\beta t)^2/(1 - \beta^2) + y'^2 + z'^2 = a^2, \tag{1}$$

where t is the observer's local time and a is a constant. This equation may be looked upon as the equation of a moving three-dimensional surface, or as the equation of a hypersurface in the hyperspace (x', y', z', t). Transforming to the system (x, y, z) we have, t being the same for both systems,

$$(lx + my + nz - c\beta t)^2/(1 - \beta^2) + (\lambda x + \mu y + \nu z)^2 + (\mathrm{L}x + \mathrm{M}y + \mathrm{N}z)^2 = a^2$$

where (l, m, n), (λ, μ, ν), $(\mathrm{L}, \mathrm{M}, \mathrm{N})$ satisfy conditions of the types

$$l^2 + \lambda^2 + \mathrm{L}^2 = 1 \text{ etc.,}$$

and

$$lm + \lambda\mu + \mathrm{LM} = 0 \text{ etc.,}$$

so that

$$\frac{\Sigma\, l^2x^2}{1-\beta^2} + \Sigma\,(1 - l^2)\, x^2 + 2\left(\frac{1}{1-\beta^2} - 1\right)\Sigma lmxy - \frac{2c\beta t\,\Sigma\, lx}{1-\beta^2} + \frac{c^2\beta^2t^2}{1-\beta^2} = a^2$$

or :

$$\Sigma\, x^2 + \frac{\beta^2}{1-\beta^2}\left\{\Sigma\, l^2x^2 + 2\,\Sigma\, lmxy - \frac{2ct\,\Sigma\, lx}{\beta} + \frac{c^2t^2}{\beta^2}\right\} - c^2t^2 = a^2,$$

which may be written

$$\phi\,(x, y, z, t) = a^2 = \text{constant}\,. \tag{2}$$

where

$$\phi\,(x, y, z, t) \equiv x^2 + y^2 + z^2 - c^2t^2 + \frac{\beta^2}{1-\beta^2}\left(lx + my + nz - \frac{ct}{\beta}\right)^2. \tag{3}$$

§ 2. Next, let W be the action function satisfying the relations :

$$p_x = \frac{\partial \mathrm{W}}{\partial x}, \quad p_y = \frac{\partial \mathrm{W}}{\partial y}, \quad p_z = \frac{\partial \mathrm{W}}{\partial z}, \quad \mathrm{H} = -\frac{\partial \mathrm{W}}{\partial t}, \tag{4}$$

where p_x, p_y, p_z are the momenta and H is the Hamiltonian function. In the simplified problem here considered these quantities are constants given by

$$\left.\begin{array}{c} p_x = m_0lc\beta/\sqrt{1-\beta^2}, \quad p_y = m_0mc\beta/\sqrt{1-\beta^2}, \quad p_z = m_0nc\beta/\sqrt{1-\beta^2} \\ \mathrm{H} = m_0c^2/\sqrt{1-\beta^2} \end{array}\right\}, \tag{5}$$

where m_0 is the proper mass of the electron, so that

$$\mathrm{W}\,(x, y, z, t) = m_0c\beta\,(lx + my + nz - ct/\beta)/\sqrt{1-\beta^2} \tag{6}$$

and on comparing (3) and (6) it is seen that

$$\phi\,(x, y, z, t) \equiv x^2 + y^2 + z^2 - c^2t^2 + \{\mathrm{W}(x, y, z, t)\}^2/m_0{}^2c^2, \tag{7}$$

Now according to Schrödinger's wave mechanics, the motion of the electron is "associated" with the propagation of a wave, the wave surfaces being given by

$$W(x, y, z, t) = \text{constant}. \tag{8}$$

Equation (7) now shows that the function $\phi(x, y, z, t)$ which was derived from the conception of the motion of a particle is built up of two functions, viz.:

$$f(x, y, z, t) \equiv x^2 + y^2 + z^2 - c^2t^2 \tag{9}$$

which corresponds to the propagation of a wave with the fundamental velocity c, and the function $W(x, y, z, t)$, which corresponds to the propagation of another wave (in this simplified case a plane wave) with the de Broglie velocity c/β. In more exact terms the hypersurface represented by equation (2) passes through the intersection of the null-cone

$$f(x, y, z, t) = 0 \tag{10}$$

and each of the two parallel hyper-planes

$$\{W(x, y, z, t)\}^2 = \text{constant} \tag{11}$$

It is not intended here to discuss the world geometry involved in this last statement as this would require a different starting point from the one that has been adopted. Relative to the material observer and his Galilean coordinates, the surface of the electron will, throughout its motion, pass through the curves of intersection of a spherical surface moving with the velocity of light and two parallel planes each moving with the velocity c/β. The "parcel" or particle aspect of the phenomenon is associated with the intersection or interference of these wave surfaces, whilst the wave aspect is associated with the wave surfaces themselves.

If one attempts to obtain the curves of intersection by solving (10) and (11) treated as simultaneous equations for a given value of t, one finds that in general, they are imaginary curves. This is due to the circumstance that the two moving planes travelling with velocity $c/\beta > c$ keep ahead of the spherical surface, itself travelling with velocity c, and hence do not intersect it in real curves. Accordingly if an observer continues to describe events in terms of his Galilean coordinates he may not discover any real connection between the particle and the wave aspects of the phenomenon. It is thus clear that a coherent account of the phenomenon can only be given in relativistic terms.

§ 3. In order to gain a fuller insight into the nature of a phenomenon it is desirable that it be viewed from different systems in relative motion. Now

there is a group of systems which stands in unique relation to all our material systems, namely, the group such that the velocity of any of its members relative to every material system is equal to the fundamental velocity c. Members of this group may be designated "radiational," in contrast with material, systems. The Lorentz transformation formula for velocities, viz.:

$$v_{13} = (v_{12} - v_{32})/(1 - v_{12}v_{32}/c^2),$$

where v_{12}, v_{32}, v_{13} are the relative velocities of three given systems, which is here used as the basis of our world geometry, permits any relative velocity between members of the *same* group except the fundamental velocity c which is itself the only permissible velocity between members of *different* groups. Thus a material particle may have any velocity (usually, however, taken to be $< c$) relative to another material particle or a ray of light relative to another ray of light, but the relative velocity between a ray of light and a particle of matter must always be c.

Returning now to the phenomenon under investigation, let it be required to view the moving electron from a "radiational" system. Equation (7) clearly shows that, since $x^2 + y^2 + z^2 - c^2t^2 = 0$ for such a system, the function $\phi(x, y, z, t)$ which described the motion of a discrete particle relative to a material system will now describe a wave phenomenon. Thus if $\phi_1(x_1, y_1, z_1, t_1)$ and $W_1(x_1, y_1, z_1, t_1)$ be the transformed functions relative to a radiational system (x_1, y_1, z_1, t_1) we have

$$\phi_1(x_1, y_1, z_1, t_1) = \{W_1(x_1, y_1, z_1, t_1)\}^2/m_0c^2. \tag{12}$$

To investigate the matter more closely let it be assumed for simplicity that $m = n = 0$, $l = -1$*, so that the motion of the electron is in the same direction relative to both systems, namely, the negative direction of the xx_1 axis, it being assumed that the velocity of the material system relative to the radiational system is in the same direction. Applying the Lorentz transformation

$$x = \operatorname*{Lt}_{u \to c} \frac{x_1 + ut_1}{\sqrt{1 - u^2/c^2}}, \qquad t = \operatorname*{Lt}_{u \to c} \frac{t_1 + ux_1/c^2}{\sqrt{1 - u^2/c^2}}, \qquad \beta = \operatorname*{Lt}_{u \to c} \frac{\beta_1 + u/c}{1 + u\beta_1/c}, \tag{13}$$

we get from (6)

$$W_1(x_1, y_1, z_1, t_1) = \operatorname*{Lt}_{\substack{u \to c \\ \beta \to 1}} \frac{-m_0c}{\sqrt{(1 - u^2/c^2)(1 - \beta^2)}} \left\{\dot{x}_1\left(1 + \frac{u}{c}\right) + t_1(c + u)\right\} \tag{14}$$

* If $l = +1$ then it is transformed into -1.

so that the equation (8) of the Schrödinger wave front becomes through this transformation

$$\operatorname*{Lt}_{u \to c} \left\{ x_1 \left(1 + \frac{u}{c}\right) + t_1 (c + u) = \operatorname*{Lt}_{\substack{u \to c \\ \beta \to 1}} \right\} \left[\text{const} \times \sqrt{\left(1 - \frac{u^2}{c^2}\right)(1 - \beta^2)} \right],$$

which on proceeding to the limit yields

$$x_1 + ct_1 = 0. \tag{15}$$

Similarly, equation (2) representing the moving surface of the electron becomes on account of (12)

$$(x_1 + ct_1)^2 = 0, \tag{16}$$

so that the *Schrödinger wave surface coincides with the surface of the electron, both reducing to a plane wave surface travelling with the velocity of light.*

It is natural to suppose that other circumstances associated with the phenomenon which are recognised by a material observer and interpreted in terms of charge, momentum, etc., will be interpreted in corresponding terms associated with a Maxwellian wave such as intensity, pressure of radiation, etc. Thus the whole of our material phenomenon as viewed by an observer travelling with the velocity of light will be described as a radiation phenomenon. Since all our material systems will have the same velocity c relative to such an observer, he will arrive at the same value for his fundamental velocity. It is tempting to inquire what interpretation will be put by a radiational observer on what a material observer describes as radiation. The most natural answer is that he will interpret them as "material" phenomena in terms of particles, charge, momentum, etc. Since a radiational system may have any velocity other than c (possibly always $< c$) relative to another radiational system it is probable that the "material" world of a radiational observer will be very similar to ours. This principle, which may be termed the principle of "reciprocal duality" between matter and radiation, possesses the advantage of completely bridging the gulf between these two fundamental entities in physical science.

Summary.

The equation representing the uniform motion of the surface of a Lorentz electron is put in a form which shows that the surface always passes through the curves of intersection of a Maxwellian wave front and two Schrödinger wave surfaces. The particle aspect of the phenomenon is associated with the interference of these two waves and the wave aspect with the propagation of the waves themselves. It is shown that from the point of view of an observer

travelling with the velocity of light, the Schrödinger wave fronts and the surface of the electron coalesce and the phenomenon thus appears as a radiation phenomenon. It is suggested that relative to such an observer our radiation phenomena will be interpreted as "material" phenomena and thus a principle of "reciprocal duality" between matter and radiation is proposed.

On the Determination of the Range of Frequencies within the Group of Mechanical Waves of an Electron.

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The wave theory of mechanics substitutes a group of waves extending over a small range of frequencies for the usual conception of the electron, thus leading to the view that the electron is a wave disturbance which travels as a compact group through space, but which does not spread into an increasingly large region like waves from a source of light or sound.

An example of this has been considered in detail by Schroedinger ('Die Naturwiss.' vol. 28, p. 664), and the group obtained by adding together the phase waves for the linear Planck oscillator.

We will consider the electronic group quite generally, without considering the special phase waves associated with it.

Let us consider the total displacement, y, arising from a group ranging in frequency from ν_1 to ν_2. Let $a_\nu\, d\nu$ denote the amplitude for the infinitesimal range $d\nu$, a_ν depending in general upon ν. The displacement corresponding to this infinitesimal range is:

$$dy = a_\nu\, d\nu \sin 2\pi\left(\nu t - \frac{\nu}{\mathrm{U}} x\right),$$

where U denotes the phase velocity and also depends on ν.

Hence

$$y = \int_{\nu_1}^{\nu_2} a_\nu \sin 2\pi\phi\, d\nu, \quad \text{where} \quad \phi = \nu t - \frac{\nu}{\mathrm{U}} x.$$

We can substitute for $d\nu$, writing $d\phi = \dfrac{\partial\phi}{\partial\nu} d\nu$, and obtain:

$$y = \int_{\phi_1}^{\phi_2} a_\nu \Big/ \frac{\partial\phi}{\partial\nu} \,.\, \sin 2\pi\phi\, d\phi.$$